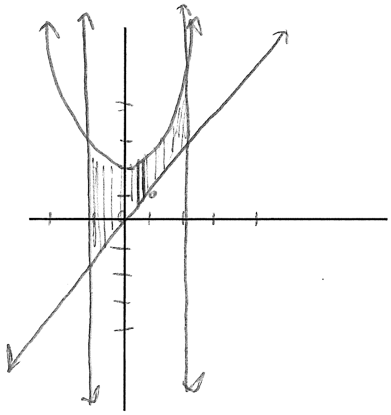
AP Calculus AB

Area Between Two Curves Worksheet #2

Name Key

Sketch the graphs, shade the bounded region and find the area bounded by the given expressions.

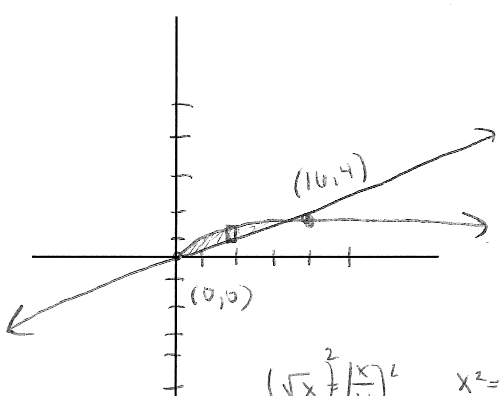
1) $y = x^2 + 1, \quad y = x, \quad x = -1, \quad and \quad x = 2$



$$\int_{-1}^{2} x^2+1-x \, dx$$

$$\frac{1}{3}x^3 + x - \frac{1}{2}x^2 \Big|_{-1}^{2}$$

2) $y = \sqrt{x} \quad and \quad y = \frac{x}{4}$



$$\int_{0}^{16} \sqrt{x} - \frac{x}{4} \, dx$$

$$\frac{2}{3}x^{3/2} + \frac{1}{8}x^2 \Big|_{0}^{16}$$

$(\sqrt{x})^2 = \left(\frac{x}{4}\right)^2$

$x = \frac{x^2}{16}$

$x^2 = 16x$

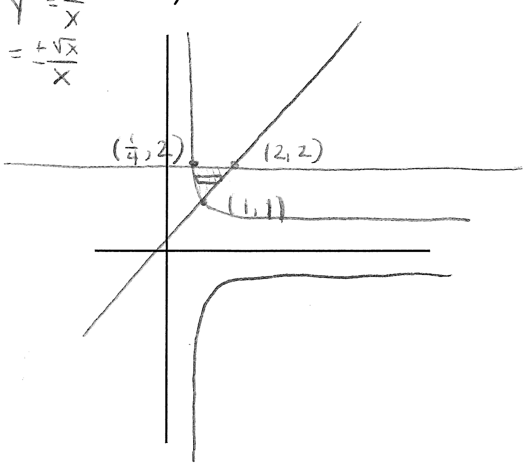
$x^2 - 16x = 0$

$x(x-16) = 0$

3) $x = \frac{1}{y^2}, \quad y = x, \quad and \quad y = 2$

$y^2 = \frac{1}{x}$

$y = \frac{\pm\sqrt{x}}{x}$



$$\int_{1}^{2} y - \frac{1}{y^2} \, dy$$

$$\frac{1}{2}y^2 + y^{-1} \Big|_{1}^{2}$$

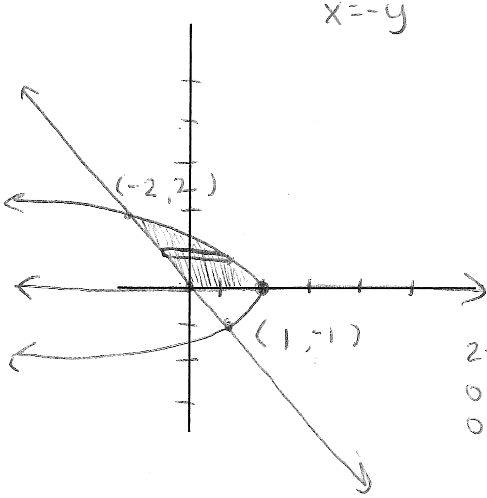
$y = \frac{1}{y^2}$

$y^2 = 1$

$y = \pm 1$

4) $x = 2 - y^2, \quad y = -x, \quad \text{and} \quad y = 0$

$x = -y$



$2 - y^2 = -y$

$0 = y^2 - y - 2$

$0 = (y-2)(y+1)$

$y = 2 \quad y = -1$

$$\int_0^2 2 - y^2 - (-y)\, dy$$

$$\int_0^2 2 - y^2 + y \, dy$$

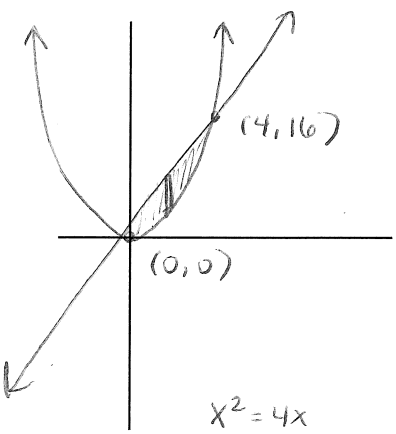
$$2y - \tfrac{1}{3}y^3 + \tfrac{1}{2}y^2 \Big|_0^2$$

$$\left[2(2) - \tfrac{1}{3}(2)^3 + \tfrac{1}{2}(2)^2\right] - [0]$$

$$\left[4 - \tfrac{8}{3} + 2\right]$$

$$\left[6 - \tfrac{8}{3}\right]$$

5) $y = x^2 \quad \text{and} \quad y = 4x$ use vertical boxes (dx)



$x^2 = 4x$

$x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0 \quad x = 4$

$$\int_0^4 4x - x^2 \, dx$$

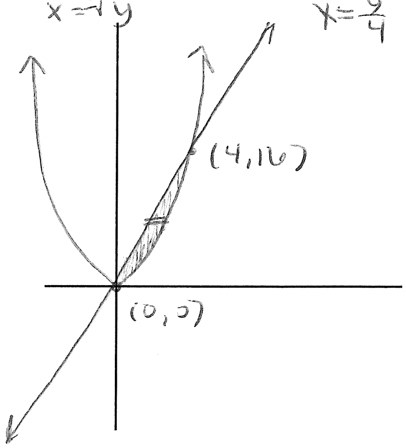
$$2x^2 - \tfrac{1}{3}x^3 \Big|_0^4$$

$$\left[2(4)^2 - \tfrac{1}{3}(4)^3\right] - [0]$$

$$\left[32 - \tfrac{64}{3}\right]$$

6) $y = x^2 \quad \text{and} \quad y = 4x$ use horizontal boxes (dy)

$x = +\sqrt{y} \qquad x = \frac{y}{4}$



$$\int_0^{16} \sqrt{y} - \frac{y}{4}\, dy$$

$$\tfrac{2}{3}y^{3/2} - \tfrac{1}{8}y^2 \Big|_0^{16}$$

$$\left[\tfrac{2}{3}(16)^{3/2} - \tfrac{1}{8}(16)^2\right] - [0]$$

$$\frac{128}{3} - \frac{256}{8}$$